Modeling the Ionospheric Delay based on Collocation for BDS Wide Area Augmentation System

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ABSTRACT

Ionospheric delay is one of the most critical errors for various applications of GNSS (Global Navigation Satellite Systems, including American GPS, Chinese BDS, Europe Galileo and Russia GLONASS). Currently, the grid-based ionospheric map broadcasted by the Wide Area Augmentation System (e.g. American WAAS, European EGNOS, Indian GAGAN, and Japanese QZSS) or released by some organizations (e.g. IGS, International GNSS Services) via internet is the major source for obtaining precise ionospheric corrections. Considering the local characteristics of ionosphere over China and its surrounding area, a new approach for precisely modeling the ionospheric delay is developed for BDS WAAS. In this approach, the ionospheric delay is considered as the sum of the trend and stochastic terms. The trend term is represented by an adjusted spherical harmonic (ASH) function, and the stochastic term is described by a time-variant covariance function. The ASH-based ionospheric model and the covariance function are simultaneously estimated using real data in order to capture the subtle variations in local ionospheric delay.

The real GPS data collected from about 30 reference stations in China were used to validate the proposed method for ionospheric modeling. The presented result shows that (1) the stochastic ionospheric delay can be depicted by the estimated covariance function and consequently improve the accuracy of grid-based ionospheric delay estimates; (2) the accuracy of...
ionospheric delay provided by the proposed approach is about 0.5m and 0.3m in the southern and northern areas of China with an average spacing of 600-1200km between permanent stations, respectively; (3) the convergence time of standard precise point positioning is shortened by about 20-30%, equivalently 8-15minutes, using the ionospheric delay correction from the proposed method.

INTRODUCTION

How to mitigate the ionospheric delay is one of the major challenges for real-time GNSS positioning no matter how many frequencies of the signals can be tracked by the GNSS receivers. The precise ionospheric correction is not only beneficial to improve the accuracy of single frequency positioning (e.g. the vast majority of navigation user), but also to boost the (re-)convergence time of dual- and trip-frequency precise positioning (e.g. precise point positioning or real-time kinematic positioning users with long baseline). The broadcast ionospheric model, the grid-based ionospheric model, the empirical ionospheric-physics model, the IGS (International GNSS Services) released Global Ionospheric Map (GIM) and some local ionospheric models are all available for real-time GNSS positioning. Among these different methods, the broadcast and grid-based ionospheric model are the most widely used in application.

The broadcast ionospheric model is distributed by the GNSS navigation message, e.g. the Klobuchar model from American GPS (IS-GPS, 2008), the Klobuchar-like model from Chinese BDS (BD-SIS-JCD, 2012) and the NeQuick model from Europe Galileo (OS-SIS-JCD, 2010). However, due to the simplicity of model structure and limitation of updating interval (1 or 2 hours), the accuracy of broadcast model is barely satisfying, especially in low latitude areas. Thus, the broadcast ionospheric model is only applicable for the navigation users achieving a positioning with a few meters accuracy.

The grid-based ionospheric map is broadcasted by the Wide Area Augmentation System (e.g. American WAAS, European EGNOS, Indian GAGAN, and Japanese QZSS) or released by some organizations (e.g. IGS). Because of the local data could be introduced for the grid-map generation and the update interval is only a few minutes (5-10 minutes), this type of model usually performs better than the broadcast ionospheric model. Thus, this paper will focus on the approach for grid-based ionospheric map generation.

Some previous researches about the ionospheric model for WAAS can be found in Kee(1993) and Enge et al.(1996). Currently, the $5^\circ$ (latitude) by $5^\circ$ (longitude) planar-fitting model is used by the American WAAS (Komjathy et al., 2002). For the Europe EGNOS, a non-homogeneous grid was designed for the ionospheric map generation and the estimation approach is basically the same as the one used by American WAAS (Arbesser-Rastburg, 2002; Loddo et al., 1996). Unlike the American WAAS and European EGNOS, the Indian GAGAN is designed for ionospheric delay correction in low latitudes where the ionospheric gradients are much larger than those in mid- or high- latitudes. Thus, a two-layer ionospheric assumption was introduced to describe the variations of ionospheric delay rather than the existing single-layer assumptions (Ratnam and Sarma, 2006a; Shukla et al., 2009). In addition, the performances of classical approaches used in American WAAS and European EGNOS are also validated in Indian area (Acharya et al., 2007; Prasad and Sarma, 2004; Prasad and Sarma, 2007). The previous achievements show that the performance of ionospheric modeling in WAAS degrades significantly with the increase of ionospheric activities in low latitudes. In order to overcome this obstacle, the correlation function and the ionospheric tomography were introduced to improve the performance of ionospheric modeling in low latitudes (Blanch et al., 2004a; b; Meza and Fernández, 2009; Sarma et al., 2009; Walter et al., 2000). The Kriging based ionospheric method is proposed for the next-generation of WAAS (Sparks et al., 2011a; b).

Since the variation of ionospheric delay generally related to local characteristics, it is difficult to obtain a ionosphere delay map with homogeneous accuracy in different areas. Therefore, the approach for modeling the ionospheric delay should be specifically designed for the WAAS located in different areas for considering the local variations of ionospheric delay.

Figure 1 shows the daily ionospheric vertical TEC during the period from 2001 (high ionospheric activities) to 2008 (low ionospheric activities) in different latitudes over China. It can be seen that the maximum value of daily ionospheric vTEC can reach about 150 TECu in the low latitudes during high ionospheric activities, while it is only about 10 TECu in the high latitudes during low ionospheric activities. These differences of ionospheric characteristics in different latitudes should be taken into account in the approach design of ionospheric modeling for BDS WAAS.
From this point of view, a new approach for modeling the ionospheric delay over China and its surrounding area is developed for BDS WAAS. Based on this proposed approach, the variations of ionospheric delay are divided into the trend and stochastic terms. An adjusted spherical harmonic (ASH) function is introduced to represent the variations of trend term, and a time-variant covariance function is proposed to describe the corresponding stochastic term. Both the ASH and covariance function are estimated using real ionospheric data from GNSS permanent stations.

The following parts of this paper are organized as follows: Firstly, the algorithm of our proposed approach will be described; and then the structure of covariance function and the estimation strategy will be shown; Thirdly, the performance of our proposed method will be validated using real data from the viewpoint of the accuracy of ionospheric delay and the convergence time of single precise point positioning; Finally, some conclusions will be drawn, as well as the future work.

**ALGORITHM**

Similarly as the traditional approach for ionospheric modeling based on GNSS data, our proposed approach is established based on the ionospheric thin-layer assumption in which the ionospheric electron is concentrated on an infinite thin layer at a height of about 450km above the earth surface. Considering the large difference of the variations of ionospheric delay in different latitudes, the variations of ionospheric delay are regarded as two parts: one is the trend term of which the ionospheric variations are smooth enough to be modeled by a mathematical function related to the latitude and longitude, and the other is the stochastic term of which the ionospheric variations are modeled by a covariance function related to the distance. The ionospheric delay at location \((\phi, \lambda)\) is represented as equation (1),

\[
I(\phi, \lambda) = \text{ASH}(\phi, \lambda) + r(x),
\]

where, \(I(\phi, \lambda)\) is the vertical ionospheric delay at the ionospheric pierce point (IPP); \(\phi\) and \(\lambda\) are the geographic latitude and longitude of corresponding IPP, respectively; \(\text{ASH}\) is the trend term of ionospheric delay, described by an adjusted spherical harmonic function shown by equation (2); \(r(x)\) is the stochastic term described by a covariance function, and \(x\) is the spherical distance between the corresponding ionospheric IPP and other neighboring IPPs; more details about the covariance function can be found in the next section.

\[
\text{ASH}(\phi, \lambda) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \mathcal{P}_n(\sin \phi') \left( \tilde{A}_n \cos(m\lambda') + \tilde{B}_n \sin(m\lambda') \right)
\]

where, \(n\) and \(m\) are the degree and order of ASH function; \(\mathcal{P}_n\) is the regularized Legendre function; \(\tilde{A}_n\) and \(\tilde{B}_n\) are the ASH coefficients to be estimated using real GNSS data; \(\phi'\) and \(\lambda'\) are the adjusted geographic latitude and longitude, respectively; the other symbols have the same meanings as that in equation (1). It should be pointed out that the distribution of ionospheric IPP is mapped to global scale before the estimation of ASH. More details about this mapping strategy can be found in Li (Li, 2012).

Following the implementation of WAAS, a set of ionospheric delay at a fixed grid point (IGP, ionospheric grid point) is calculated from real ionospheric delay. Figure 2 shows a sketch of the geometric relationship between the IPPs and IGP. Assuming that there are \(N\) ionospheric IPPs around one IGP in the coverage area, i.e. in the circle with a radius of \(c\), the observed ionospheric delay at each IPP and the estimated ionospheric delay at each IGP can be written as equation (3).

![Figure 2 Sketch of the geometric relationship between the IPPs and IGP](Image)

\[
\begin{align*}
L_i + V_i &= A_i X + Y_i \\
L_j &= A_j X + Y_j
\end{align*}
\]

where, \(L_i\) is the vector consisting of the observed ionospheric delay in the coverage area; \(V_i\) is the vector of observation error and its variance-covariance matrix is assumed as \(\mathbf{D}_{i} \); \(A_i\) is the design matrix related to the distribution of ionospheric IPPs; \(X\) is the vector of estimated coefficients of ASH; \(Y_i\) is the vector of stochastic ionospheric delay at each ionospheric IPP and its variance-covariance matrix is assumed as \(\mathbf{D}_{y} \); \(V_i\) and \(Y_i\) are mutually independent; \(L_i\) is the estimated ionosphere delay at IGP; \(A_i\) is the design matrix related to the location of IGP and has the same formation with \(A_i\); \(Y_i\) is the estimated stochastic ionospheric delay at IGP.

Based on the solution of collocation proposed by Zhou (Jiangwen, 2002), equation (3) can be solved following two steps:

**Step1:** Rewrite the observation equation (3) in the form shown by equation (4).
where, $L_{1\alpha}$ can be regarded as the observation of $A_iX$, and $V_{1\alpha}$ is the corresponding observation error. Since $V_i$ is independent of $Y_i$, the variance-covariance matrix of $V_{1\alpha}$ can be written as equation (5).

$$D_{V_{1\alpha}} = D_{V_i} + D_{V_j}$$  \hspace{1cm} (5)

Thus, the $X$ can be estimated by the least squares estimation, showing as in equation (6).

$$\hat{X} = N_{1\alpha}^T A_i^T D_1 V_{1\alpha}, N_{1\alpha} = A_i^T D_1^T A_i$$ \hspace{1cm} (6)

Although there is no $Y_i$ in the observation equation (4), but the stochastic information of $Y_i$ has been considered in equation (5). Thus, the $\hat{X}$ in equation (6) is the most optimal estimation.

Step2: The equation (3) can be transformed to equation (7) with the known estimates of $\hat{X}$.

$$\begin{align*}
L_{1\beta} + V_{1\beta} &= Y_{1\beta} \\
V_{1\beta} &= L_{1\beta} - A_i\hat{X} \\
D_{V_{1\beta}} &= N_{1\beta}^T A_i^T D_{V_{1\alpha}} A_i
\end{align*}$$ \hspace{1cm} (7)

where, $L_{1\beta}$ can also be considered as the observation of $Y_{1\beta}$; $V_{1\beta}$ is considered as the corresponding observation error; thus, the variance-covariance matrix of $V_{1\beta}$ is shown as in equation (8).

$$\begin{align*}
D_{V_{1\beta}} &= R_i D_i + J_i D_i J_i^T \\
J_i &= A_i N_{1\beta}^T A_i D_{V_{1\alpha}} A_i \\
R_i &= I - J_i
\end{align*}$$ \hspace{1cm} (8)

Based on the least squares estimation, the estimates of $Y_{1\beta}$, i.e. $Y_i$, can be obtained from equation (9).

$$\begin{align*}
L_{1\gamma} &= L_{1\beta} - A_i\hat{X} \\
D_{V_i} &= R_i D_i + J_i D_i J_i^T + J_i D_i J_i^T
\end{align*}$$ \hspace{1cm} (9)

Now, the optimal estimates of $X$ and $Y_i$ has been obtained, and then the estimates of ionospheric delay at IGP can be calculated by equation (10) and (11).

In addition, it should be pointed that the errors of vertical ionospheric delay at each gird point, i.e. GIVE, Grid Ionoospheric Vertical Error, which is indispensable for WAAS, but it beyond this contribution.

$$\begin{align*}
\hat{Y}_i &= D_{V_{1\gamma}} D_{V_{1\gamma}}^T \hat{X}_1 = D_{V_{1\gamma}} D_{V_{1\gamma}}^T R_i L_i
\end{align*}$$ \hspace{1cm} (10)

ESTABLISHMENT OF COVARIANCE FUNCTION

The spherical function is used to represent the covariance of ionospheric delay, as shown by equation (12) and (13) (Olea, 1999).

$$C(d) = \gamma(\varphi) - \gamma(d)$$ \hspace{1cm} (12)

$$\gamma(d) = \begin{cases} 
\frac{c_\varphi}{2} (3d - d^3) & 0 \leq d \leq R \\
\frac{c_\varphi + c}{R} & d > R 
\end{cases}$$ \hspace{1cm} (13)

where, $C(d)$ and $\gamma(d)$ are the covariance and variogram function of ionospheric delay at two locations with the spherical distance of $d$, respectively; $\gamma(\varphi)$ is the variogram of ionospheric delay at two locations with infinity distance; $c_\varphi$, $c$ and $R$ are the unknown parameters of variogram function and need to be estimated using real data.

Since the variations of ionospheric delay are significantly different at different levels of ionospheric activities, the variogram function shown by equation (13) is determined using the observed ionospheric delay at all IPPs in a real-time mode and match the real situation as much as possible. The stochastic term of ionospheric delay is estimated based on equation (9). Figure 3 illustrates an example of estimated covariance and variogram function of ionospheric delay at different levels of ionospheric activities.

![Figure 3 Covariance and variogram function estimated using real data at different levels of ionospheric activities](image)

It can be found that the differences of variogram are very large at different levels of ionospheric activities, thus it is impossible to use a unified model to describe the covariance of ionospheric delay. However, the covariance function in some previous proposed approaches was generally considered as a fixed function (Blanch et al., 2004a; Ratnam and Sarma, 2006b; Sparks et al., 2011b). Different from those existing approaches, a real-time covariance function is introduced by the proposed approach and it could adaptively capture the subtle variation of ionospheric delay.
DATE USED FOR PERFORMANCE VALIDATION

Because there is no public BDS data from a reference network in China at present, the GPS data is used for the performance validation in this section. Since the ionosphere is independent from the navigation system, the GPS data is also able to validate the accuracy of the proposed approach for ionospheric modeling to some extent.

Three experiments were carried out in different levels of ionospheric activities for this validation. The sunspots number during the period of data collection is shown by Figure 4. It can be seen that the solar activities during the year of 2002, 2008 and 2012 are in low, high and middle levels. The data in each experiment was collected from the 76th -83rd day of the corresponding year.

The experimental data was collected from Crustal Movement Observation Network of China (CMONC) and there were 21 reference stations contributed to our experiment during the period of high and low ionospheric activities (2002 and 2008), while there were 29 reference stations during the period of middle ionospheric activities (2012). In each experiment, 8 stations located in different latitudes were selected as rovers to validate the accuracy of ionospheric delay predicted from the proposed approach. The data from those rover stations was not used in the ionospheric modeling. The distributions of reference and rover stations are given by Figure 5 and Figure 6.

The line-of-sight (LOS) ionospheric delay is extracted from the dual-frequency code and phase observation at the reference stations. The technique of phase leveling code is used to improve the accuracy of extracted ionospheric delay (Li et al., 2014). The satellite and receiver DCB were directly corrected by the estimates obtained on the previous day and they were determined using a two-step method proposed by Li et al. (2012). The modified mapping function was used to transfer the LOS ionospheric delay to the vertical direction at the IPP (Schaer, 2008). A 15-minute moving window was selected for the ionospheric modeling in real-time mode, and an elevation-depended weight function was used, and the cut-off elevation was set as 15° to ultimately reduce the multipath error.

An ionospheric delay map is generated at a set of fixed grid and broadcasted to the rover by the communication link in WAAS. Considering the variations of ionospheric delay at different latitudes, an inhomogeneous grid is designed for BDS WAAS over China and its surrounding area. The coverage and resolution of ionospheric map is shown by Figure 7. The intervals of latitudes are 8° at high latitudes (greater than 42°N), 4° at middle latitudes (from 34°N to 42°N) and 2° at low latitudes (lower than 34°N) (Li, 2012). The vertical ionospheric delay of each satellite at rover can be interpolated from a set of ionospheric delays at those four surrounding grid points and then converted to the LOS direction by the modified mapping function (Schaer and Werner, 1998).
ACCURACY OF IONOSPHERIC DELAY

The real LOS ionospheric TEC is obtained from the raw dual-frequency data of rover station and converted to the vertical direction at ionospheric IPP. The accuracy of proposed approach can be validated by comparing the ionospheric TEC from the grid-based ionospheric map with that from raw dual-frequency data. The difference of ionospheric delay for each epoch is calculated following equation (14).

\[
\Delta I_i = \frac{I_i - I_i' - Bias_i}{MF_i'} \\
Bias_i = \frac{\sum_{j=1}^{n} (I_i' - I_i)}{n}
\]

where, \( I_i' \) is predicted LOS ionospheric delay from the grid-based ionospheric map for satellite \( j \) at epoch \( i \); \( I_i' \) is the LOS ionospheric delay from the raw dual-frequency data at rover station for satellite \( j \) at epoch \( i \); \( MF_i' \) is the modified mapping function for satellite \( j \) at epoch \( i \); \( n \) is the number of tracked satellite at epoch \( i \).

The DCB of rover receiver denoted by \( Bias_i \) in equation (14) is difficult to be calibrated; thus, only the DCB of satellite is corrected in the real ionospheric TEC calculation. However, the DCB of receiver can be aborted by the estimates of receiver clock bias in positioning. \( \Delta I_i \) can be used to represent the accuracy of predicted vertical ionospheric delay \( I_i' \).

Four rover stations (ULAB, URUM, BJFS and ZHNZ) located in middle latitudes and the other rover stations are in low latitudes. The differences of ionospheric delay obtained at those rover stations are shown in Figure 8 and Figure 9. The black, blue and pink dots represent the differences at high (2002), low (2008) and middle (2012) ionospheric activities.

It can be found that the accuracy of predicted ionospheric delay mainly depends on the levels of ionospheric activities. The accuracy during the period of high ionospheric activities (2002) is much worse than that during the period of low ionospheric activities (2008). The daily-cycle variations are obvious in the differences in low latitudes no matter of the level of ionospheric activities. The reason is that the variations of ionospheric delay in low latitudes are much larger than that in middle latitudes, as shown in Figure 1.

Based on the distribution of reference stations and the grid map of ionospheric delay broadcasted in our experiment, the accuracies of predicted ionospheric delay in middle latitudes are better than 0.4m and 0.2m in high and low ionospheric activities periods respectively; however, they are about 0.7m and 0.2m in low latitudes, respectively.
VALIATION BY PRECISE POINT POSITIONING

The convergence time of dual-frequency PPP is generally about 20 minutes, and the precise ionospheric delay is positive to accelerate the convergence process of PPP. In this section, the ionospheric delay from WAAS broadcasted ionospheric map is used as a pseudo-observation and its priori variance is calculated following equation (15).

\[
\sigma_{\text{ion},k}^2 = \alpha_k \cdot \sigma_{\text{ion}}^2 \cdot MF(\varepsilon)
\]  

(15)

where, \(\sigma_{\text{ion},k}^2\) is the priori standard deviation of predicted ionospheric delay at frequency \(k\); \(\sigma_{\text{ion}}^2\) is the priori standard deviation of vertical ionospheric delay at the first frequency; \(MF(\varepsilon)\) is the modified mapping function depending on the elevation; \(\alpha_k\) is the factor to transfer the variance from the first frequency to the frequency \(k\). \(\sigma_{\text{ion}}^2\) is empirically set as 0.3m and 0.5m for the rover in middle and low latitudes, respectively.

The PPP Kalman filter is reinitialized every two hours. The stations URUM, ZHNZ, QION, BJFS and ZHNZ are selected as the rovers for analyzing the convergence time when adding the predicted ionospheric delay as pseudo-observation. The precise satellite orbit and clock from IGS are used in the PPP solution. The PPP observation model is established directly using the raw observation. The PPP Kalman filter is reinitialized every two hours.

Figure 10 and 11 illustrate the positioning accuracy with and without the predicted ionospheric delay as pseudo-observation at the 79th day of year 2008 and 2012, respectively. The coordinates estimated based on the static and post-processing PPP are considered as true values. Due to the limitation of space, only the results during some periods are shown in Figure 10 and 11.

It can be found that the positioning accuracy in the initialization stage is improved significantly with the constraint of predicted ionospheric delay. During the period of low ionospheric activities, the positioning accuracy of traditional PPP at the first epoch is about 3.0-4.0m, even about 10m in the low latitudes; however, it is improved to about 1.0m with the predicted ionospheric delay. During the period of middle ionospheric activities, the accuracy at initialization stage can be improved from 1.5-2.0m to about 1.0m. The positioning result with and without the predicted ionospheric delay are closer to each other with data accumulation.

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delay in year 2008 and 2012, respectively. The convergence times for horizontal and vertical direction are individually calculated. The last column shows the average convergence time of those five rover stations.

During the period of low ionospheric activities, the convergence times on horizontal and vertical direction are about 3000s and 2000s without the predicted ionospheric delay, whereas they are only about 2000s and 1100s when using the constraint of predicted ionospheric delay. Different from the result during the period of low ionospheric activities, the convergence time can only be improved from about 2000s and 1600s to about 1600s and 1200s in horizontal and vertical direction during the middle ionospheric activities. The reason for shortening the convergence time is that more observation information, i.e. the predicted ionospheric delay, can be introduced during the positioning estimation and it makes the model stronger and can reach the required accuracy level faster.

In order to show a quantitative improvement for the constraint of predicted ionospheric delay, an index is defined as equation(16).

\[
mp = \frac{Sec_{\text{Addion}} - Sec_{\text{NoIon}}}{Sec_{\text{NoIon}}} \times 100\% 
\]

Where, \(mp\) indicates the improvement percent for the constraint of predicted ionospheric delay on convergence time; \(Sec_{\text{Addion}}\) and \(Sec_{\text{NoIon}}\) are the average convergence times with and without predicted ionospheric delay during one day.

The improvement percentage denoted by \(mp\) is shown in Table 1 and 2. The convergence times for the horizontal and vertical direction are individually calculated. The required accuracies in the horizontal and vertical are 0.2m, respectively. It can be seen that (1) the improvement in low latitudes is more significant than that in middle latitudes; (2) the improvement percentage is about 30%-40% (about 15 minutes) when using the predicted ionospheric delay during the experimental period of year 2008, whereas it is only about 20%-30% (about 8 minutes) during the experimental period of year 2012. The reason is that the accuracy of grid-based ionospheric map in middle latitudes is much better than that in low latitudes (see Figure 8 and 9).

However, the result for low ionospheric activities is different from that in high and middle ionospheric activities, e.g. the convergence time for the rover in low latitudes is improved more significantly than the one in middle latitudes. It is because (1) the accuracy of grid-based ionospheric map in different latitudes are comparable during the period of low ionospheric activities, and (2) the empirical standard deviation cannot reflect the real accuracy of predicted ionospheric delay. The strategy of using the predicted ionospheric delay as a pseudo-observation for PPP user is only preliminary, how to obtain the prior variance is very crucial and deserves further consideration.

Table 1 Comparison of the convergence time of different rover stations with and without the predicted ionospheric delay constraint on the 79th day of year 2008 (unit: sec)

<table>
<thead>
<tr>
<th>Strategy*</th>
<th>BJFS</th>
<th>URUM</th>
<th>ZHNZ</th>
<th>WUHN</th>
<th>QION</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convergence time for the required horizontal accuracy</td>
<td>Nolon</td>
<td>3120</td>
<td>3025</td>
<td>2965</td>
<td>3227</td>
<td>2845</td>
</tr>
<tr>
<td>Addon</td>
<td>2724</td>
<td>2181</td>
<td>1950</td>
<td>1585</td>
<td>1397</td>
<td>1967</td>
</tr>
<tr>
<td>(mp)</td>
<td>12.7%</td>
<td>27.9%</td>
<td>34.2%</td>
<td>50.9%</td>
<td>50.9%</td>
<td>35.3%</td>
</tr>
<tr>
<td>Convergence time for the required vertical accuracy</td>
<td>Nolon</td>
<td>966</td>
<td>1932</td>
<td>2275</td>
<td>2322</td>
<td>2498</td>
</tr>
<tr>
<td>Addon</td>
<td>876</td>
<td>1405</td>
<td>1330</td>
<td>698</td>
<td>1245</td>
<td>1111</td>
</tr>
<tr>
<td>(mp)</td>
<td>9.3%</td>
<td>27.3%</td>
<td>41.5%</td>
<td>69.9%</td>
<td>50.2%</td>
<td>39.6%</td>
</tr>
</tbody>
</table>

*Nolon: Without predicted ionospheric delay constraint; Addon: With predicted ionospheric delay constraint.

Table 2 Comparison of the convergence time of different rover station when adding the predicted ionospheric delay constraint on the day of year 79th in 2012 (unit: sec)

<table>
<thead>
<tr>
<th>Strategy*</th>
<th>BJFS</th>
<th>URUM</th>
<th>ZHNZ</th>
<th>JXHK</th>
<th>QION</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convergence time for the required horizontal accuracy</td>
<td>Nolon</td>
<td>1360</td>
<td>3043</td>
<td>2027</td>
<td>1911</td>
<td>1753</td>
</tr>
<tr>
<td>Addon</td>
<td>910</td>
<td>2254</td>
<td>1150</td>
<td>1697</td>
<td>1260</td>
<td>1454</td>
</tr>
<tr>
<td>Improved</td>
<td>33.1%</td>
<td>25.9%</td>
<td>43.3%</td>
<td>11.2%</td>
<td>28.1%</td>
<td>28.3%</td>
</tr>
<tr>
<td>Convergence time for the required vertical accuracy</td>
<td>Nolon</td>
<td>965</td>
<td>2199</td>
<td>1850</td>
<td>1521</td>
<td>1586</td>
</tr>
<tr>
<td>Addon</td>
<td>685</td>
<td>1213</td>
<td>1347</td>
<td>1320</td>
<td>1534</td>
<td>1220</td>
</tr>
<tr>
<td>Improved</td>
<td>29.0%</td>
<td>44.8%</td>
<td>27.2%</td>
<td>13.2%</td>
<td>3.3%</td>
<td>23.5%</td>
</tr>
</tbody>
</table>

*Nolon: Without predicted ionospheric delay constraint; Addon: With predicted ionospheric delay constraint.
CONCLUSION AND FUTURE WORK

The ionospheric delay map is one of the essential augmentation information broadcasted by WAAS. Since the ionosphere is intrinsically of local characteristics in different areas, it is impossible to establish a unified approach to generate the ionospheric delay map for the WAAS in different areas. In this contribution, an ionospheric modeling approach for the BDS WAAS over China and its surrounding areas was proposed. Based on the proposed approach, the variations of ionospheric delay are modeled as a combination of trend and stochastic terms. The trend term of ionospheric delay is represented by an adjusted spherical harmonic function, and the stochastic term is described by a covariance function in the form of sphere function. The coefficients of the adjusted spherical harmonic function and the covariance function are all determined using the ionospheric delay from real data based on the collocation estimation. Considering the different variations of ionospheric delay at different latitudes, an inhomogeneous grid is proposed for BDS WAAS over China and its surrounding area.

Three sets of real GPS data individually collected in different ionospheric activities (2002, 2008 and 2012) from about 30 reference stations in China were used to validate the performance of the proposed approach. Different from the existing approach, the time-variation covariance function is able to capture the subtle variations of ionospheric delay more precisely. With the inter-station distance of 600-1200km, the accuracy of ionospheric delay map is about 0.3m and 0.5m over the northern and southern areas of China. The predicted ionospheric delay is able to accelerate the convergence process of PPP and could reduce the convergence time by about 20-30% (8-15minutes). The experimental result shows that the proposed approach can not only be used for BDS WAAS in China, but also applicable to other low latitudes for the ionospheric modeling.

Due to the lack of open access BDS data in China, the performance of our proposed approach is only validated by the GPS data at present at this stage; however, considering the difference between GPS and BDS on the constellation and observation noise, the proposed approach also needs to be further validated with real BDS data. Moreover, the approach of calculating GIVE still need further studied to make the GIVE more confidence.

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